Factoring Quadratics

A quadratic equation is a polynomial of the form $ax^2 + bx + c$, where *a*, *b*, and *c* are constant values called *coefficients*. You may notice that the highest power of *x* in the equation above is x^2. A quadratic equation in the form $ax^2 + bx + c$ can be rewritten as a product of two factors called the "factored form". This form resembles (x + ?)(x + ?) and is useful in determining the x-intercepts of a parabola, the graph of a quadratic equation. Factoring a quadratic polynomial can be frustrating, but the techniques described below may provide some assistance.

The Simple Case ($a = 1$)

For the simple case, $a = 1$, you will find two numbers that multiply to equal *c*, and add together to equal *b*. For example:

- Factor $x^2 + 5x + 6$

We need to find factors of 6 (the *c* term) that add up to 5 (the *b* term). Since 6 can be written as the product of 2 and 3, and since 2 + 3 = 5, we'll use 2 and 3. This quadratic is formed from multiplying the two factors (x + 2)(x + 3).

- If *c* is positive, the signs of both factors are the same as the *b* term.
- If *c* is negative, the signs of both factors are opposites, and the largest factor is the same sign as the *b* term.

The Not As Simple Case ($a \neq 1$)

The **Box Method** can be used to factor quadratics, including the simple case, but it is very useful when $a \neq 1$. Be sure to have the quadratic in its simplest form.

- Factor $4x^2 + 4x - 15$

1) Make a box and divide it into 4 squares:

2) Put the ax^2 term in the top left corner:

$4x^2$	

3) Put the *c* term in the bottom right corner:

$4x^2$	
	-15

4) Multiply both terms (ax^2 and c) → generate a list of factors.

$4x^2 \bullet -15 = -60x^2 \rightarrow$	$\pm 2x$,	$\pm 30x$
	$3x$,	$20x$
	$4x$,	$15x$
	$5x$,	$12x$
	$6x$,	$10x$

5) Determine which sum or difference of these factors will give you the bx term:

$$bx = 4x \rightarrow -6x+10x = 4x$$

6) Put these terms in the remaining boxes:

$4x^2$	$-6x$
$10x$	-15

7) Factor out common terms to the outside of the box:

$2x$	-3	
$4x^2$	$-6x$	$2x$
$10x$	-15	5

8) Read factors from the sides of the box:
$(2x - 3)(2x + 5)$

9) Double check—each box will be a product of the outside terms.

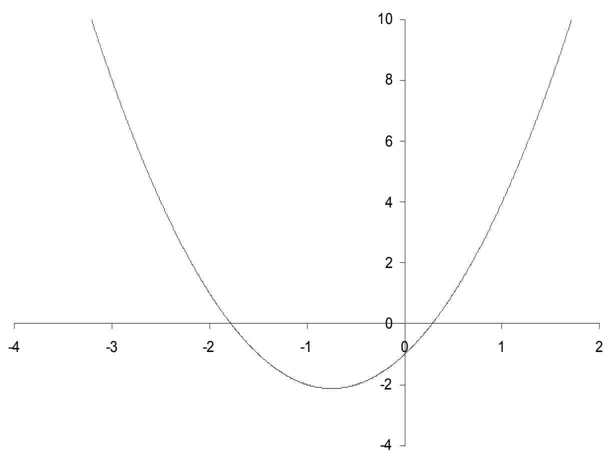
The Quadratic Formula

When we have a quadratic equation in the form $ax^2 + bx + c = 0$, we can find the value(s) of x where y = 0 by plugging the coefficients (a, b, and c) into this formula to solve the equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula can be used for any quadratic equation in the form $ax^2 + bx + c = 0$ and will yield solutions that need not be whole numbers.

For example: $2x^2 + 3x - 1 = 0$; $a = 2$, $b = 3$, $c = {}^{-}1$



$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm \sqrt{17}}{4} \approx {}^{-}1.78, 0.28$$

Note that the equation gives two values for x. These values represent the x-intercepts. You may also notice that the quadratic formula contains the expression $b^2 - 4ac$ under the square root. This expression is called the discriminant. The discriminant is an indicator of how many real solutions are possible. If the discriminant is greater than 0 (positive), there are two real solutions. If the discriminant is equal to 0, there is one real solution. If the discriminant is less than 0 (negative), there are no real solutions.